

Learning Difficulties Australia

Learning Difficulties Australia is an association of teachers and other professionals dedicated to assisting students with learning difficulties through effective teaching practices based on scientific research.



www.ldaustralia.org



enquiries@ldaustralia.org



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Nathalie Parry

MEd (Specific Learning Difficulties)

Joint PhD Candidate (Melbourne Graduate School of Education
& Katholieke Universiteit Leuven, Belgium)

- Specialist teacher in the area of learning differences, lecturer in initial teacher education and learning intervention.
- Current research focuses on the learning potential of young children at risk of mathematics learning difficulties through interactive learning

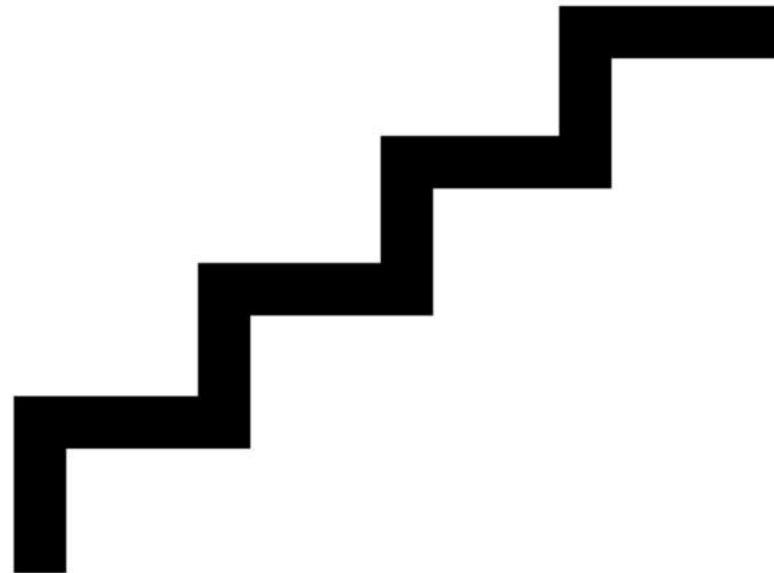


Importance of Mathematics

- Early mathematical skills are strong predictors of later mathematics achievement (See Aunola et al., 2004; Duncan et al., 2007; Geary et al., 2013; Jordan et al., 2009)
- Early mathematics skills more accurately predict later reading abilities than precursors of reading (Claessens et al., 2009; Duncan et al., 2007)



- Future acquisition of mathematical skills is facilitated by early numerical skills (e.g., Aunola et al., 2004; Jordan et al., 2009)





Areas of importance

- Number & operations
- Geometry
- Spatial sense,
- Measurement
- Patterning
- Algebraic thinking
- Displaying & analysing data

- We need to concentrate on early mathematics skills that are the most relevant for early interventions (Aunio & Räsänen, 2015)
- Focus on numeracy and operations of number—the domain most predictive of later mathematics achievement (Nguyen et al., 2016).



- Children who start school behind their peers in mathematics are at a greater risk of remaining so unless they receive targeted intervention (e.g., Duncan et al., 2007)



Early Numerical Competencies

- Counting
- Number patterns
- Place value
- Symbols
- Number recognition
- Quantity discrimination
- Addition & subtraction concepts
- grouping of 10
- Basic number combinations
- Mental number line



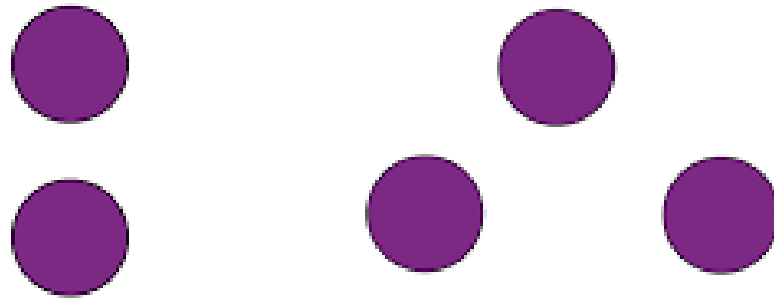
Counting

Counting encompasses five principles:

- Stable order,
- One-to-one correspondence,
- Cardinality,
- Abstraction, and
- Order irrelevance (Gelman & Gallistel, 1978).



- Subitizing is the ability to instantly recognise how many items are in a group.
- Students who struggle with mathematics also struggle with subitizing (Schleifer & Landerl, 2011)



Comparing Numbers



4

9



Students with difficulty in mathematics often struggle with comparing numbers and perform lower on comparing tasks than peers without mathematics difficulties (De Smedt & Gilmore, 2011)

Instead of "how many?"...

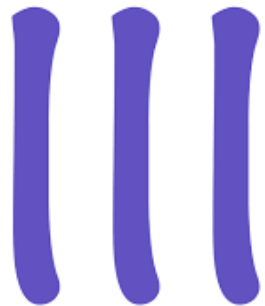
Ask "How many fewer? Or Which has more?"



Symbols

Mathematical symbols are important because most of mathematics is represented using symbols.

“three”



3

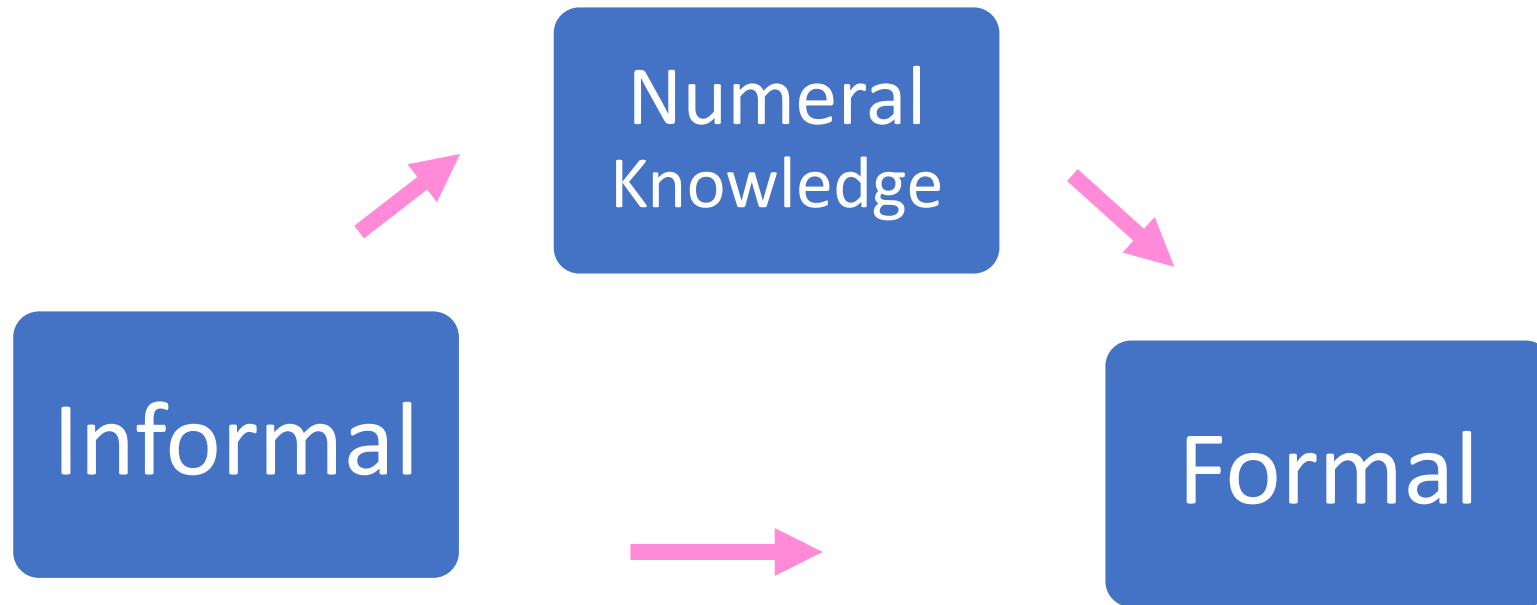


Addition & Subtraction

- The mastery of counting, comparing numbers, and mathematical symbols does not necessarily come after learning the principles of addition and subtraction.
- Children can solve simple addition and subtraction problems presented without symbols (i.e., presented orally and/or with manipulatives or counting)
- Children's addition skills are generally stronger than subtraction skills- and why students may solve subtraction problems using addition (Torbeyns et al., 2009)



Informal- formal numeracy



(Purpura, Baroody, Lonigan 2013)



Informal numeracy knowledge

- Informal numeracy knowledge include skills learnt prior to, and outside of, formal schooling
- Involves connecting quantities to number words flexibly and understanding the relations among quantities.



- Foundation - comparison of sets, recite number word sequence
- Meaningful numbering skills - 1:1 correspondence.
Cardinality, subitising
- Operations on verbal numbers - e.g. adding means quantity increases



Number knowledge

- Before children can apply their informal knowledge in the formal domain, they must map that knowledge onto the Arabic numeral system (Purpura et al., 2013)
- Differentiate between numerals and other signs/symbols (e.g., numerals and letters are different) and learn names of numerals
- Numeral knowledge has been found to be a strong, (if not the strongest), predictor of later formal mathematics ability, probably because it is a precursor for acquiring formal numeracy knowledge.



- Children with maths difficulties have specific difficulties with the symbolic numerical system rather than informal
- A deficit in aspects of numeral knowledge may inhibit children's successful acquisition of formal maths.

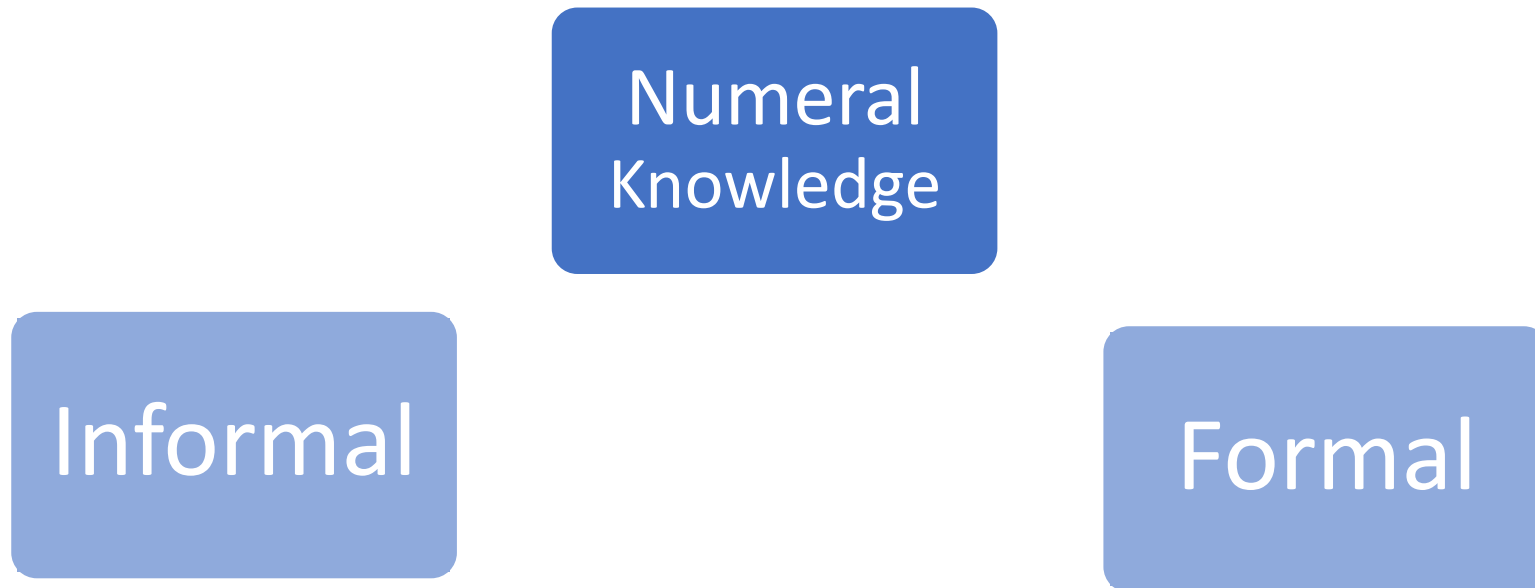


Formal numeracy knowledge

- Formal numeracy knowledge consists of the mathematical concepts and skills that children learn in school, through formal instruction.
- Eg: Arabic numerals, equality and operation signs, base ten



Although children's formal mathematical skills have been shown to be malleable through targeted interventions (Fuchs et al., 2009), the mechanism by which the transition from informal to formal knowledge occurs is unclear.



(Purpura, Baroody, Lonigan 2013)



Language & literacy

- Phonological awareness- related to quantity comparison and number word sequence
- Linguistic skills may be necessary for early math terminology
- Numeral and letter ID



Mathematical language

- Mathematical language is a child's understanding of the key words and concepts used in early math.
- Mathematical language consists of terms that are used to describe **quantity or spatial relations** (e.g., more, fewer, near, below).



Quantitative Language

- “More” “less” “fewer”
- Describe and make comparisons
- Refine understanding of quantity



Spatial Language

- “Before” “after” “next” “between”
- Describe dimension of objects, location, direction
- Relation to physical objects , numberline



Mathematical Language

- The strongest and most consistent classifier of the children who performed the lowest on a numeracy measure at the end of preschool was *mathematical language*
- Mathematical language was found to be a better classifier than beginning of preschool numeracy performance (Purpura et al., 2017)



Supporting early mathematics

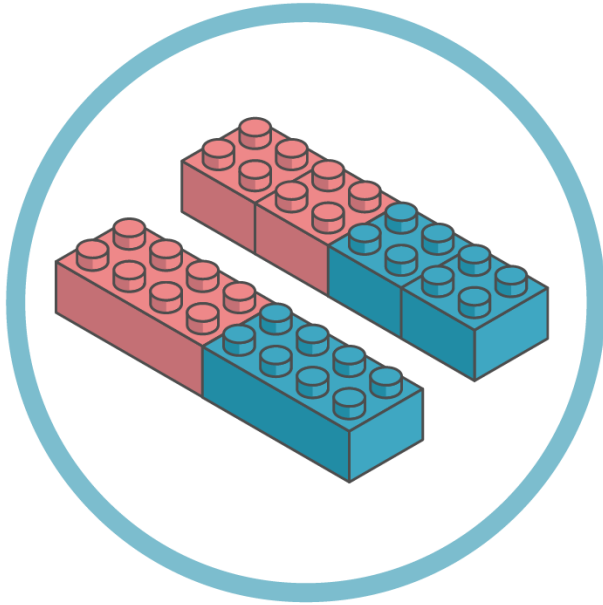
- Clements and Sarama (2011) suggest effective intervention programs include a comprehensive set of cognitive concepts and processes aligned with known mathematical developmental sequences, and a mix of instructional methods including explicit instruction.
- E.g. learning trajectories include three components: a goal, a developmental progression, and instructional activities.



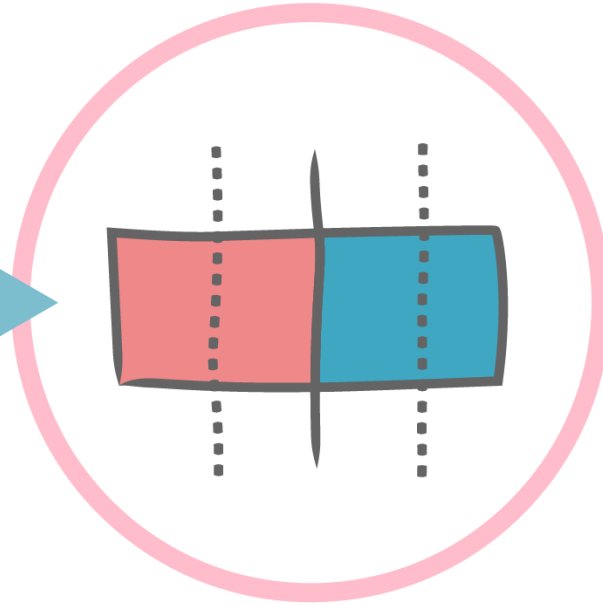
- Intervention are effective for basic math skills
- Direct instruction
- Studies show significant improvement in outcomes using the concrete-representational-abstract sequence (Kroesbergen & Van Luit, 2003)



Concrete



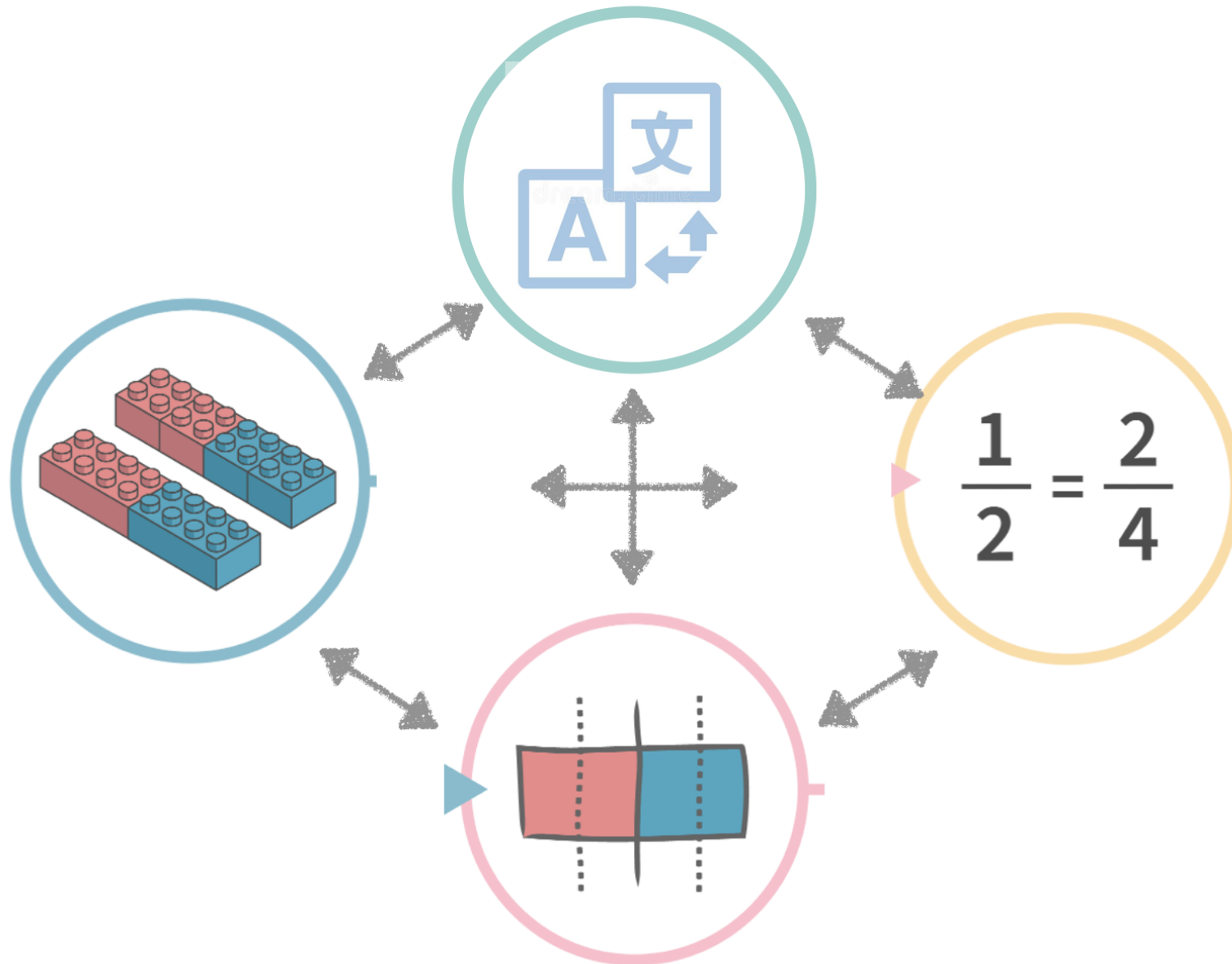
Representational



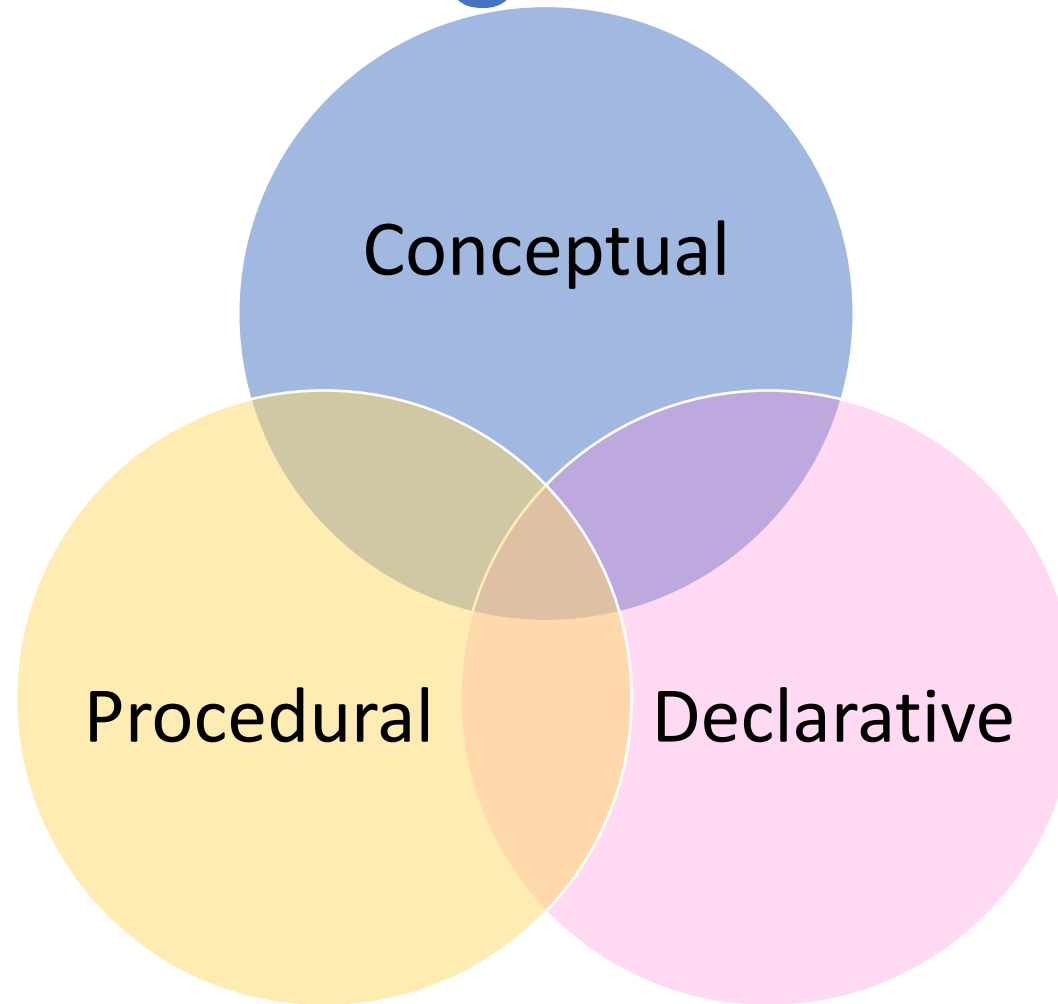
Abstract

An orange circle containing the mathematical equation $\frac{1}{2} = \frac{2}{4}$.





Types of knowledge (Miller & Hudson, 2007)



Supporting early mathematics at home

“To bolster children’s mathematics development, we should encourage the use of mathematics skills in the home *and* the classroom—and just as early, with just as much emphasis, as we do reading” (Purpura, Litkowski & Knopik, 2019. p. 19).



Example Layout From the First Book of the Series: *Too Many Pillows*

"I packed a lot of pillows," Benjamin said. "I don't want the ground to be hard."

"Bear and I packed pillows too!" said Lucy.

Bear had a pillow. Lucy had more pillows. Benjamin had the most pillows.

1. Who has more pillows: Bear or Benjamin?

2. Why doesn't Bear need as many pillows as Lucy or Benjamin?

3. Benjamin likes to sleep with a lot of pillows. What do you like to sleep with?



Note. From *The Little Elephants' Big Adventures: Too Many Pillows*, by A. Isaacs and M. Dye (Illustrator), with the Purdue Early Achievement Research Labs, 2021. Copyright 2021 by David J. Purpura, Sara A. Schmitt, and the Trustees of Purdue University. See the online article for the color version of this figure.

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