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Core foundations of early mathematics: refining the number sense framework

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Preschool number sense can be operationalized as three interconnected strands — number, number relations, and number operations. These strands involve key ideas that are foundational to mathematics education. Recent cognitive and behavioral research refines and extends our understanding of the early number sense framework in the following ways: (1) Although number sense can be viewed as a single construct, each strand predicts achievement when controlling for the others, and the strands appear to reinforce each other during development; (2) Level of representation (i.e. nonsymbolic versus symbolic) and set size affect children's competencies and development within and across strands and should be considered in intervention research; (3) There are substantial individual differences in preschoolers' number sense knowledge. We argue that instruction must weave together these number sense strands from the start of preschool to prepare children for success in formal mathematics.

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Introduction

Although the importance of early schooling is widely recognized, support for early mathematics historically received less attention than support for early literacy [1]. About a decade ago, the Committee on Early Childhood Mathematics of the National Academy of Sciences [2] synthesized the rapidly growing body of research on early mathematical learning with the aim of transforming approaches to teaching young children mathematics. One key recommendation stated that early math experiences should concentrate on developing core number sense competencies, which includes three strands: number, number relations, and number operations. These core competencies allow children to work with exact, rather than approximate, representations of number [3].

Not surprisingly, early number sense predicts later mathematics achievement, when controlling for cognitive, behavioral, and demographic factors [4,5]. Moreover, number sense is malleable. That is, most preschoolers can acquire number sense competencies through instruction [6]. The number sense framework provides important targets for screening and intervention [7]. The NRC committee challenged traditional notions about the mathematics curriculum in early childhood, which often was incidental to general classroom activities, such as calendar time, and focused on rote counting and numeral recognition without emphasizing foundational concepts. Instead, the Committee argued that early number competencies involve essential mathematical ideas that merit dedicated instruction in preschool and kindergarten, which in turn help children succeed with formal mathematics [4].

Elements of the number sense framework

Early *number* involves skills related to knowledge of whole numbers and the counting list, such as one-to-one correspondence, cardinality, and recognizing numerals (See Box 1 for a glossary of terms) [8,9]. For example, children begin to recite the count list verbally. Children also learn to count each object only once in a stable order and that the final number in the sequence indicates how many items are in the set (i.e. the cardinality principle). Knowledge of the cardinality principle is often assessed through a task in which children are asked to produce a sub-set with a specific cardinal value from a larger set of objects, commonly referred to as the 'Give a number' or 'Give-N' task. *Number relations* involves knowledge of the relations between and among whole numbers (e.g. determining which quantity is more, less, or equal to another quantity) [10]. By engaging with number relations, children form preliminary understandings that numbers in the count list are represented linearly, with each number being exactly one more than the previous one [11,12], allowing them to order numbers and estimate their placement on a number line. Facility with *number operations* involves knowledge of how whole numbers can be taken apart and put together. Specific

Box 1 Glossary

Number sense: foundational number competencies that develop during the early years, operationalized as knowledge of number, number relations, and number operations with exact quantities.

Cardinality principle: a number strand understanding that the last number word said when counting a set of objects indicates the quantity of the set.

Subitizing: the ability to quickly ascertain the quantity of a nonsymbolic small set (four or less) without counting.

Level of representation: the way a quantity is presented to a child, including *nonsymbolic* (represented without using symbols as pictures or objects) and *symbolic* (represented with spoken number words or written numerals).

Set size: the quantity being presented to a child, including *small* (usually within the subitizable range of 1–4) and *large* (> 4).

Give-N (give-a-number task): commonly used to assess an understanding of the cardinality principle, where the child is asked to produce a subset with a specific cardinal value from a larger set of objects.

Magnitude comparison: a task commonly used to assess number relations, in which a child is asked to choose which of two *nonsymbolic* dot arrays or *symbolic* numerals is larger.

Nonverbal arithmetic: a task used to assess early number operations, in which a child is shown a small set of objects that is then covered and transformed by adding or taking away one or more objects before the child indicates how many are now under the cover.

skills include composing and decomposing quantities (e.g. four can be broken down into sets of three and one and two and two) [13]. Children learn they can transform exact quantities through adding or subtracting, whether problems are presented nonverbally, as stories, or as symbolic number combinations [14••,15]. The three core number sense strands typically follow a developmental path during preschool and kindergarten [6,14••], which is influenced by children's early experiences at home and in school [16,17,18].

In recent years, cognitive and behavioral research has extended our understanding of the *interconnectedness* of number, number relations, number operations, and the developmental paths the strands follow:

1. Findings reveal that although number sense can be viewed as a single construct, each strand predicts achievement when controlling for the others, and the strands appear to reinforce each other.
2. Studies emphasize the importance of level of representation (i.e. nonsymbolic versus symbolic) and set size in understanding children's competencies within and across strands.
3. There are individual differences in children's number sense knowledge in early childhood that should be addressed in instruction.

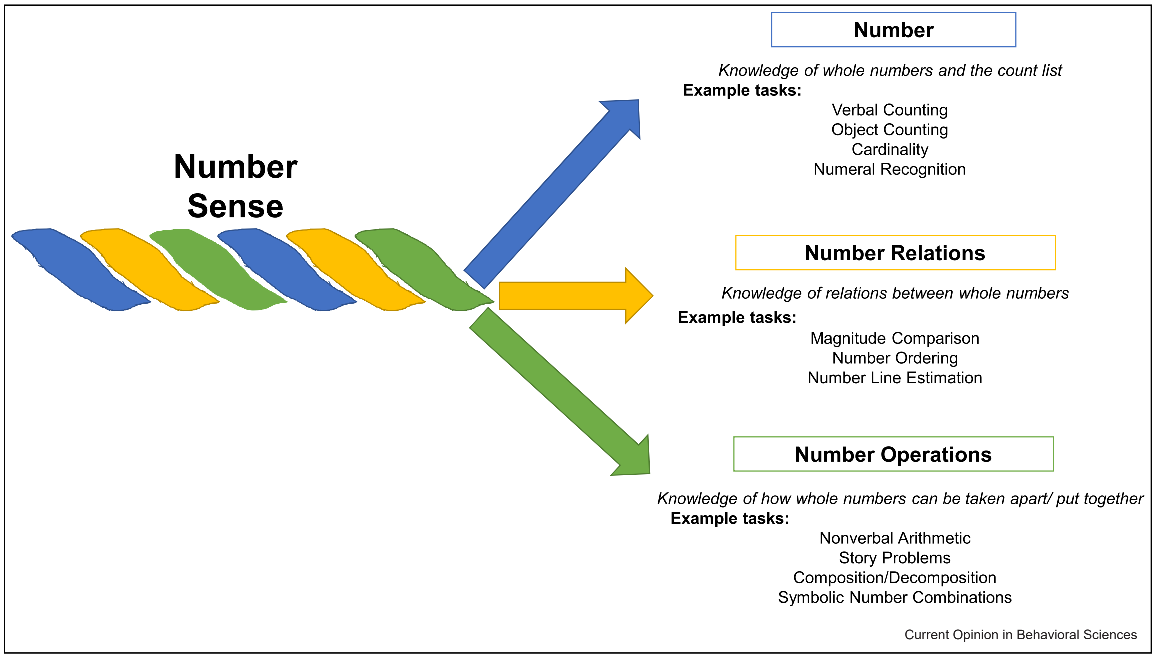
In this review, we discuss current studies related to these topics and how they refine our understanding of the number sense framework. We argue that number sense strands must be intertwined, like strands of a rope, as children develop key foundations of early mathematics (See Figure 1). Despite the flurry of research activity on early number knowledge and development in the last decade, there are areas within the number sense framework requiring investigation. We identify these areas to focus future research on continued refinement of the framework.

Number sense strands are connected but relatively unique predictors of mathematics achievement

Recent studies examining the structure of preschool mathematics provide clear empirical evidence of a multifactor number sense model, consisting of number, number relations, and number operations [19•,20]. Factor analysis also reveals a single number sense dimension that is relatively distinct from other topics in early mathematics [19•], such as geometry and measurement, although the latter topics also involve applications of number sense knowledge. Specific subdomain knowledge can be fostered through specific early number activities [21].

A recent study [22••] examined the relative importance of number, number relations, and number operations for predicting mathematics achievement one year later, while accounting for the other strands as well as age, income-level, gender, and multilingual learner status. Preschoolers ($N = 150$, $M_{age} = 4.5$ years) were given the *Screener for Early Sense* [23], a reliable measure of preschool number sense knowledge across the three strands. The researchers used both ordinary least squares (OLS) regression, used to investigate average relations among predictors and outcomes, and quantile regression, which allows for the exploration of whether predictive relations are inconsistent across different quantiles of the outcome distribution. Using OLS regression analyses, the researchers showed each strand uniquely contributed to kindergarten mathematics achievement a year later, with overall number sense accounting for 74% of the variance in general mathematics achievement. Effect sizes were 0.40, 0.33, and 0.20 for number, number relations, and

Figure 1



The number sense framework.

number operations, respectively. Quantile regression analyses were then conducted to predict kindergarten mathematics achievement at low (0.2), intermediate (0.5) and high (0.8) quantiles of the entire kindergarten mathematics achievement distribution. Analyses showed predictability of the number sense strands differed according to achievement quantile. When controlling for other variables, the number strand explained a significant amount of variance in mathematics achievement at the low and intermediate quantiles but not at the high quantile; number relations explained a significant amount of variance in mathematics achievement at all the tested quantiles; and number operations explained a significant amount of variance in kindergarten mathematics achievement at the two higher but not at the low quantile. Results for high achievers indicated that children who grasp basic aspects of number (e.g. counting and cardinality) and number relations (e.g. magnitude comparison; see Box 1) can leverage these skills to engage in number operations.

Although this work confirms the existence of distinct but highly related strands of early number sense and their importance for later mathematics achievement, there are still topics requiring further investigation. For example, research must test the relative contributions of the strands of early number sense on varied topics in later mathematics learning, such as geometry. Future work should also continue to disentangle the three strands and how they relate to one another as they develop over time. One recent study has pointed to a developmental sequence of early number sense learning in preschool, with specific number relations skills serving as a potential intermediary between number and number operations competencies [24]. This finding is in line with work suggesting that knowledge of the cardinality principle usually develops before specific number relations abilities [25]. However, these studies were conducted with cross-sectional data, and longitudinal models are needed to test a developmental sequence that moves from number, to number relations, to number operations. Training studies are also needed to test predictions of causal relations between subdomains. It is possible that the development of the strands of number sense is iterative, whereby this sequence repeats and strengthens with increasing task demands.

Level of representation and set size matter in the development of number sense

An important consideration when thinking about numerical development is the *level of quantity representation* (i.e. nonsymbolic versus symbolic), which affects children's reasoning about numbers across the three strands. One study [26••] examined connections among preschoolers' knowledge of nonsymbolic exact quantities (e.g. four shapes), spoken number words (e.g. the word 'four'), and numerals or digits (e.g. 4). In a sample of 2–4-years-old children ($N = 68$; M_{age} = 3.4 years), the researchers found that children's knowledge of nonsymbolic exact quantities, spoken number words, and numerals was associated with their ability to map between symbolic and nonsymbolic quantities (e.g. to name and identify the numeral that represents a picture of four rectangles). Mapping from quantities to written numerals developed after mapping to spoken number words. Importantly, this mapping knowledge predicted children's skill with symbolic magnitude comparisons across single-digit set sizes (e.g. which is more — 3 or 4?), a number relations skill. The study showed that written numeral to quantity mapping is critical to learning more complex tasks related to number relations and operations.

Symbolic and nonsymbolic processing of numbers, however, must be considered within *set sizes*. In a recent longitudinal study, researchers [27••] examined 4–6-year-olds' ($N = 540$; $M_{age} = 5.17$) magnitude comparison skills on nonsymbolic (which of two sets of dots has more, without counting) and symbolic (which of two numerals is bigger) tasks. Trials for both types of presentations were further categorized into comparisons of small sets of one to four and large sets of five to nine. It was found that symbolic and nonsymbolic magnitude comparison skills were more strongly related for small set sizes, which are in the subitizable range — where children can apprehend exact quantities instantly — than for larger sets, which may be processed approximately. Considering growth across the school year, while controlling for performance in the fall, symbolic and nonsymbolic comparison with small sets predicted positive change in the other condition over the course of the school year (i.e. a bidirectional relation). However, for larger sets outside of the subitizing range of one to four, there was evidence of a unidirectional relation whereby start of year symbolic comparison was a significantly stronger predictor of growth in nonsymbolic comparison rather than vice versa. Knowledge of number symbols seems to facilitate skill with estimation of larger numbers, perhaps through fast implicit counting [28,29].

A key implication is that nonsymbolic quantity representations scaffold development of symbolic understanding, but only for small sets. For small sets, nonsymbolic and symbolic numbers may be processed similarly. It was previously shown that children recognize and discriminate between small exact quantities without counting at a very early age, before they learn number words [30]. That is, small number quantities and numerals both involve exact representation whereas larger quantities require children to estimate nonsymbolic quantities. The findings suggest that for younger children, activities should first be presented with small sets using both nonsymbolic and symbolic presentations and then progress to larger sets as more efficient counting skills are established. This suggestion is in keeping with earlier work showing that children as young as three years of age can solve exact calculations with small sets on nonsymbolic, nonverbal arithmetic tasks by forming a mental model [31]. For example, the child is shown three dots, which are then covered; the examiner adds one more dot under the cover and the child indicates how many dots are under the cover. Preschoolers give exact verbal responses (e.g. the number word four) to nonverbal problems as easily as they can put out the exact number of dots or identify pictures of the correct number of dots [32]. Similarly, pictorial representations on a Give-N cardinality task with preschoolers produced performance like and sometimes better than performance on a parallel task using physical manipulatives [33]. Unfortunately, not all researchers consider the variables of set size and level of representation in their studies. Additional work must consider how set size and level of representation may constrain the development of number sense skills within strands and how training children on concepts in one set size or level of representation may affect performance in another set size or level of representation.

There are individual differences in the development of number sense

It is well established that preschool through kindergarten is a period of extensive numerical development [14••]. However, as development involves the integration of multiple number sense concepts across representations and set sizes, it is unsurprising that not all children follow the same developmental pathway. Recent work has captured variation in preschoolers' number sense understanding and development across the strands using person-centered approaches. Traditional variable-centered approaches assume the relation between number sense variables can be applied to all children, while person-centered approaches aim to uncover distinct profiles of children who share similarities in specific patterns of skills. Person-centered approaches can be extended to longitudinal data through methods such as latent transition analyses, which can be used to determine whether individual children show the same latent profiles over time.

A longitudinal study [34•] used latent transition analyses to track numerical competencies of 128 preschoolers (M_{age} = 4.0 years) as they transitioned from preschool to kindergarten. Skills assessed were primarily symbolic, including numeral recognition (3–20), cardinal number knowledge (Give-*N*), number word to quantity mapping without counting, and ordering sets of three single-digit numerals from smallest to largest. Children's skills were tracked at three time points over an 8-month span. The goal was to identify profiles of children based on development of multicomponent number sense competencies. Results revealed distinct developmental pathways. A key result was that children who were consistently low across the study period displayed little growth on basic cardinality tasks, which may also have affected their development of numeral ordering. In contrast, children in a low to intermediate performance group made relatively strong gains in cardinality knowledge across time points. These findings reinforce the idea that number knowledge (cardinality) helps children engage effectively in other number skills, but training studies are needed to test this prediction.

Another study [35•] explored variability in a sample of preschoolers from low-income families ($N = 115$, $M_{age} = 4.6$ years). Although low-income children clearly represent a high-risk group for later mathematics difficulties [36,37], their numerical development has often been treated with broad strokes. The researchers assessed skills representative of number (e.g. counting, cardinality, and numeral recognition) and number relations (e.g. nonsymbolic and symbolic magnitude comparison) strands. Supporting previous results that included a range of income levels [34•,38], there was evidence of empirically distinct profiles, including overall poor number sense; overall strong number sense; overall moderate number sense; and strong numerical skills, including cardinality, with relatively weaker knowledge of numerical magnitudes. The presence of the last group seems to contrast with the work [34•], which suggested the importance of cardinality knowledge for understanding relations between numbers; however, growth across the year was not examined, and it is possible that children in this group would begin to use their cardinality knowledge to build knowledge of numerical magnitudes. It should also be noted that the two studies used different set sizes and levels of representation, and neither study analyzed the data specifically along these dimensions. Moreover, the studies did not examine knowledge of number operations, which also develop during this period [14••].

Individual differences in the development of number sense are related to other child-level cognitive variables, such as executive functions [34•,35•] and language [35•]. Contextual factors, including children's home and school numeracy environments, are also related to number skill and development [21,39,40]. Research has begun to explore number sense abilities and growth in dual language learners and found that early number sense knowledge assessed in both languages was correlated [41]. Future work should continue to explore individual differences in early number sense using person-centered approaches with cross-cultural and diverse samples.

Conclusions and implications for practice

Recognizing the importance of early mathematics, researchers continue to refine our understanding of how number sense strands build on each other during development. Although strands within the framework represent relatively separate dimensions, we argue that number, number relations, and number operations knowledge must be woven together right from the start of preschool instruction. This model has motivated the development of early screening tools to identify students at risk for mathematics difficulties and disabilities [23,42]. A recent study [14••] also used the number sense framework to outline developmental trajectories for specific early number sense skills across the three strands. Using cross-sectional, item-level data from 801 preschoolers, the authors charted learning trajectories for skills representative of the number, number relations, and number operations. Importantly, the authors also considered set-size and level of representation in the developmental trajectories. Such fine-grained analyses, based in the number sense framework, are essential for aligning learning standards and activities with children's knowledge in preschool classrooms, where instructional time is often focused on literacy over mathematics concepts [14••].

Existing studies, though not conclusive, suggest the interesting possibility that young children can engage in number (e.g. counting and cardinality), number relations (e.g. magnitude comparisons) and number operations (e.g. addition and subtraction) activities within the subitizable range of small quantities from one to four. Children might learn to count and represent these small quantities with number words and then numerals, compare exact quantities with small sets, and engage in exact calculations with objects present. An intervention approach based on this model has been successful with low performing kindergartners in randomized studies [7]. For example, children build numbers from one to three quickly with blocks or their fingers, learn to recognize their cardinal values with a visual cardinality chart, sequence spoken numbers and numerals from one to three, decompose sets of three objects into partners of two and one, and solve nonverbal, story-based, and symbolic number combinations with totals up to three. As children develop skills with small numbers, they gradually cycle to a similar sequence of activities with larger quantities with 10. Future research should examine the promise of this type of training approach for younger children, at the start of preschool. Given sizeable individual differences in preschool numerical development [14••,34•,35•], such an approach will pinpoint where a child's individual strengths and weaknesses lie within the number sense framework and lead to targeted interventions.

Conflict of interest statement

There are no conflicts of interest related to this article.

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This study presents descriptive statistics to create a map of developmental trajectories of number competencies across the preschool years. The authors argue that rigorous research on the development of specific number competencies is needed to help introduce numerical concepts to the preschool classroom, where math content is currently sparse and misaligned to children's abilities. Importantly, the authors consider task context and set size in their fine-grained trajectories.

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